Nombres complexes

Correction

Exercice 1

Rappels : $e^{ix} = \cos x + i\sin x$

$e^{-ix} = \cos x - \sin x = \overline{e^{ix}}$

$\cos x = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)$

$\sin x = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)$

Formule de Moivre :

$\left(e^{ix}\right)^n = \left(\cos x + i\sin x\right)^n = e^{inx} = \cos nx + i\sin nx$

$$A = \sin^3 x \cos^2 x$$

$$A = \left(\frac{e^{ix} - e^{-ix}}{2i}\right)^3\left(\frac{e^{ix} + e^{-ix}}{2}\right)^2$$

$$A = \frac{1}{i^3}\frac{1}{2^5}\left(e^{3ix} - 3e^{ix} + 3e^{-ix} - e^{-3ix}\right)\left(e^{2ix} + 2 + e^{-2ix}\right)$$

$$A = \frac{i}{2^5}\left(e^{5ix} + 2e^{3ix} + e^{ix} - 3e^{3ix} - 6e^{ix} - 3e^{-ix} + 3e^{ix} + 6e^{-ix} + 3e^{-3ix} - e^{-ix} - 2e^{-3ix} - e^{-5ix}\right)$$

$$A = \frac{i}{2^5}\left[\left(e^{5ix} - e^{-5ix}\right) - \left(e^{3ix} - e^{-3ix}\right) - 2\left(e^{ix} - e^{-ix}\right)\right]$$

$$A = \frac{i}{2^5} \cdot 2i\left(\sin 5x - \sin 3x - 2\sin x\right)$$

$$A = \frac{1}{16}\left(2\sin x + \sin 3x - \sin 5x\right)$$